AIChE

Control of Spatially Distributed Processes with Unknown Transport-Reaction Parameters via Two Layer System Adaptations

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DOI 10.1002/aic.14852
Published online May 12, 2015 in Wiley Online Library (wileyonlinelibrary.com)

The control problem of dissipative distributed parameter systems described by semilinear parabolic partial differential equations with unknown parameters and its application to transport-reaction chemical processes is considered. The infinite dimensional modal representation of such systems can be partitioned into finite dimensional slow and infinite dimensional fast and stable subsystems. A combination of a model order reduction approach and a Lyapunov-based adaptive control technique is used to address the control issues in the presence of unknown parameters of the system. Galerkin's method is used to reduce the infinite dimensional description of the system; we apply adaptive proper orthogonal decomposition (APOD) to initiate and recursively revise the set of empirical basis functions needed in Galerkin's method to construct switching reduced order models. The effectiveness of the proposed APOD-based adaptive control approach is successfully illustrated on temperature regulation in a catalytic chemical reactor in the presence of unknown transport and reaction parameters. © 2015 American Institute of Chemical Engineers *AIChE J*, 61: 2497–2507, 2015
Keywords: distributed parameter systems, adaptive model reduction, adaptive control, transport-reaction process, process control

Introduction

Dissipative distributed parameter systems (DPS) naturally appear in chemical process industries due to reactions and transport phenomena mechanisms (diffusion, convection, and dispersion) which are the source of nonlinear behavior and spatial variations, respectively. Such chemical processes, exemplified by tubular reactors, packed-bed reactors, and chemical vapor deposition systems, can be mathematically modeled by a set of semilinear parabolic partial differential equations (PDEs) when diffusive mechanisms dominate over convection and dispersion.[1]

In aforesaid processes where an interplay between reaction and momentum, energy and mass transport phenomena exists, physical parameters, such as Reynolds, Rayleigh, Prandtl, Peclet numbers, and reaction parameters of the system are often unknown and must be identified. This fact shows the importance of using system identification and adaptive strategies in monitoring and control of such processes.[2,3] While adaptive control of finite-dimensional systems is an advanced field that has produced adaptive control methods for a very general class of linear time-invariant systems,[4–6] system identification and adaptive control techniques have been developed for only a few classes of PDEs restricted by relative degree, stability, and domain wide actuation assumptions.[7–14] There are two sources of difficulty in dealing with PDEs with parametric uncertainties. The first difficulty, which also exists in ordinary differential equations (ODEs), is that even for linear plants, adaptive schemes are nonlinear. The second issue, unique to PDEs, is the absence of parameterized families of controllers. In this manuscript, our objective is to circumvent the difficulties of adaptive control synthesis for semilinear parabolic PDE systems via model order reduction (MOR).

The spatiotemporal dynamics of semilinear parabolic PDEs can be approximated by reduced finite dimensional models because the eigenspectrum of spatial differential operators of such systems can be decomposed into a finite dimensional slow and probably unstable, and infinite dimensional fast and stable subsets.[1] The traditional approach that takes advantage of the aforesaid partitioning property to address the control problem of semilinear parabolic PDEs is via MOR.[15–19] MOR applies spatial discretization techniques like Galerkin's method to convert the continuous operator problem (in the form of PDE) to a discrete problem (proper number of ODEs). The resulting set of ODEs is called a reduced order model (ROM), which captures the dominant dynamics of the original PDE system and may be used as the basis to synthesize the controller. The bottleneck in this procedure is the computation of the minimum number of proper basis functions which are needed in Galerkin's method to construct the ROM. In simple linear cases, the basis functions can be analytically computed from the eigenproblem of spatial differential operators of parabolic PDEs. However, the applicability of such analytical methods is limited because most of the systems in the chemical process

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industries include complex spatial dynamics and have irregular process domains. Therefore, even for linear distributed parameter systems in their general form, analytical approaches may be inapplicable to construct the ROM.

To bypass this limitation, statistical principle component techniques like proper orthogonal decomposition (POD) are used to compute the required basis functions empirically.[20–24] However, POD requires the priori availability of a sufficiently large ensemble of PDE solution data in which the most prevalent spatial modes are excited. Recently, to circumvent this requirement an efficient recursive computation algorithm was developed. It is known as adaptive proper orthogonal decomposition (APOD), and is used as additional data from the process becomes available. APOD is based on algebraic manipulations leading to a threefold increase in computational speed compared to similar real time optimization-based techniques.[23] A modification to APOD based on information theory concepts was introduced to regulate the distributed parameter systems with fast transients.[21] The continuous measurement sensors requirements were then reduced using APOD-based dynamic observers.[25,26] Furthermore, a set of criteria was identified to minimize the frequency of collecting snapshots considering closed-loop stability.[27] An APOD-based geometric observer/controller pair was also synthesized to track the desired output of nonlinear dissipative distributed parameter systems.[28,29]

Even though adaptation laws have been previously used to revise POD techniques online in the literature,[21,23,30–35] to the best knowledge of the authors, the combination of data-driven model reduction and adaptive controller synthesis have not been previously applied to address the control problem of DPS with unknown parameters. The numerous control structures that are based on APOD and other POD-like methods that can be found in the literature, cannot be used directly to address such problems because the controllers cannot be constructed in the presence of unknown parameters in the system. Thus, we applied a combination of adaptive control strategy and APOD to address the technical issues related to control based on model reduction in the presence of unknown parameters while the system parameters have a significant effect on the MOR approaches.

In this article, an adaptive output feedback controller is synthesized to regulate the dissipative distributed parameter systems, described by semilinear parabolic PDEs, in the presence of unknown parameters. APOD is applied to construct and recursively update the ROM which is the basis for the synthesis of control structure. The preliminaries section introduces the studied semilinear parabolic PDE and its infinite dimensional representation in an appropriate Sobolev subspace. A short review on APOD algorithm and Galerkin's method is also included for completeness. An application to thermal dynamics regulation in a catalytic chemical reactor via APOD-based adaptive control design is finally presented. It includes a detailed controller synthesis discussion and an illustration of the effectiveness of the proposed MOR and control strategies.

Preliminaries

A class of semilinear parabolic PDE system

We consider processes modeled by semilinear parabolic PDE systems in the following state space form

$$\frac{\partial}{\partial t}\bar{x}(z,t) = \mathbf{A}\frac{\partial^2}{\partial z^2}\bar{x}(z,t)+\mathbf{f}(\bar{x}(z,t),\theta(t))+b(z)u(t) \quad (1)$$

subject to boundary and initial conditions

$$q\left(\bar{x},\frac{\partial \bar{x}}{\partial z}\right)=0 \;\text{ on }\; \partial\Omega, \quad \bar{x}(z,0)=\bar{x}_0(z) \quad (2)$$

In the PDE system of (1) and (2), $z \in \Omega \subset \mathbb{R}^3$ denotes the spatial coordinate, t is the time, Ω is the process domain with boundary $\partial\Omega$. $\bar{x}(z,t) \in \mathbb{R}$ stands for the state variable and $u(t) \in \mathbb{R}^l$ is the manipulated inputs vector. $\mathbf{A}$ is a positive constant and $\mathbf{f}(\cdot)$ is a sufficiently smooth nonlinear vector function. $\theta(t)$ is the vector of unknown parameters and $b^T(z) \in \mathbb{R}^l$ is a smooth matrix function which describes the control action distribution in the spatial domain, for example, point actuation is described by a standard Dirac delta. $q(\cdot)$ is a nonlinear vector function and $\bar{x}_0(z)$ is a smooth vector function.

The availability of two types of measurement sensors is assumed; periodic distributed snapshot measurements, $y_r(z, t_k) \in \mathbb{R}$ that indicates measured spatial profiles, and continuous measurements, $y_m \in \mathbb{R}^r$ that is a vector variable

$$\begin{aligned} y_m(t) &= \int_\Omega s(z)\bar{x}(z,t)\, dz \\ y_r(z,t_k) &= \int_0^t \delta(\tau-t_k)\bar{x}(z,\tau)\, d\tau \end{aligned} \quad (3)$$

where r is the number of continuous sensors, t_k is the time instance for snapshot measurement, and $k=1,2,\ldots$. The function $s(z)$ denotes the continuous point sensors distribution in the process domain, Ω, and $\delta(\cdot)$ indicate standard Dirac delta function.

Remark 1. The results of this manuscript are presented for $\bar{x} \in \mathbb{R}$, however, we may directly extend the results for $\bar{x}(z,t) = [\bar{x}_1(z,t) \cdots \bar{x}_n(z,t)]^T \in \mathbb{R}^n$ by treating each state individually. The interactions between distributed system states is then captured through the spatial integration of the respective basis functions with $\mathbf{f}(\bar{x},\theta)$ to give appropriate nonlinear functions.[24]

System representation in Sobolev subspace

The semilinear parabolic PDE system of (1)–(3) can be presented as an infinite-dimensional system in a relevant Sobolev subspace of $\mathbb{W}^{1,2}(\Omega,\mathbb{R})$, that satisfies the homogeneous boundary conditions of (2), that is

$$\mathbb{W}^{1,2}(\Omega,\mathbb{R}) = \left\{F \in \mathbf{W}^{1,2}(\Omega,\mathbb{R}) : q\left(F,\frac{\partial F}{\partial z}\right)=0 \;\text{ on }\; \partial\Omega\right\}$$

where $\forall i,j \in \mathbb{N}$, $i \geq 1$ and $1 \leq j < \infty$, Sobolev space of

$$\mathbf{W}^{i,j}(\Omega,\mathbb{R}) = \{\bar{x} \in L_j(\Omega) : \partial^\alpha \bar{x} \in L_j(\Omega), \forall \alpha \in \mathbb{N}, |\alpha| \leq i\}$$

is a functional space that includes all of the α-order differentiable functions with respect to all of spatial coordinates of the process domain. We can also define the inner product and norm since $\mathbb{W}^{1,2}$ belongs to the space of square integrable functions, $L_2(\Omega)$, in the following form

$$(\vartheta_1,\vartheta_2) = \int_\Omega r(z)\vartheta_1^T(z)\vartheta_2(z)dz, \quad \|\vartheta_1\|_2 = (\vartheta_1,\vartheta_1)^{1/2}$$

where ϑ^T denotes the transpose of ϑ and $r(z)$ is the weight function that is assumed to be 1 in this work. To simplify the notation, we will use $\mathbb{W}$ to denote $\mathbb{W}^{1,2}(\Omega,\mathbb{R})$ for the remainder of the article.

On the basis of the above, we define within $\mathbb{W}$ the state

$$x(t)=\bar{x}(\cdot,t),\ x\in\mathbb{W}^{1,2} \tag{4}$$

the linear and nonlinear differential operators

$$\mathcal{A}x(t)=\mathbf{A}\frac{\partial^2}{\partial z^2}\bar{x}(\cdot,t),\quad \mathcal{F}(x(t),\theta(t))=\mathbf{f}(\bar{x}(\cdot,t),\theta(t)) \tag{5}$$

the manipulated input operator

$$\mathcal{B}u(t)=b(z)u(t) \tag{6}$$

and measured outputs' operators

$$\mathcal{S}x(t)=(s^T(z),\bar{x}(\cdot,t)),\ \mathcal{R}x(t)=\int_0^t\delta(\tau-t_k)\bar{x}(\cdot,\tau)\,d\tau \tag{7}$$

Then, the PDE system of (1)–(3) can be presented in the Sobolev subspace as follows

$$\begin{aligned}\dot{x}(t)&=\mathcal{A}x(t)+\mathcal{F}(x(t),\theta(t))+\mathcal{B}u(t),\ x(0)=x_0\\ y_m(t)&=\mathcal{S}x(t)\\ y_r(t_k)&=\mathcal{R}x(t_k)\end{aligned} \tag{8}$$

where $k=1,2,\ldots$, for Eqs. 7 and 8. The set of analytical basis functions of the system, needed to build the ROM, can in principle be obtained from the solution of the eigenvalue problem for the linear spatial operator of the system as follows

$$\mathcal{A}\phi_i=\lambda_i\phi_i,\quad i=1,\ldots,\infty \tag{9}$$

subject to the boundary conditions

$$q\left(\phi_i,\frac{d\phi_i}{dz}\right)=0\ \text{ on }\ \partial\Omega \tag{10}$$

where λ_i and ϕ_i denote the ith eigenvalue and the corresponding basis function, respectively.

Assumption 1. *We assume that the ordered eigenspectrum of* $\mathcal{A}$, $\sigma_A=\{\lambda_1,\lambda_2,\ldots\}$, *can be partitioned into a finite set of* r_s *slow eigenvalues,* $\sigma_{\mathcal{A}}^{(s)}=\{\lambda_1,\lambda_2,\ldots,\lambda_{r_s}\}$, *and a countable complement set of the remaining fast eigenvalues* $\sigma_{\mathcal{A}}^{(f)}=\{\lambda_{r_s+1},\lambda_{r_s+2},\ldots\}$. *The associated basis functions sets are defined as* $\Phi_s=[\phi_1\ \phi_2\ \cdots\ \phi_{r_s}]^T$, $\Phi_f=[\phi_{r_s+1}\ \phi_{r_s+2}\ \cdots]^T$. *There is a large separation between the slow and fast eigenvalues of A, that is,* $|Re(\lambda_1)|/|Re(\lambda_{r_s})|=O(1)$ *and* $|Re(\lambda_1)|/|Re(\lambda_{r_s+1})|=O(\varepsilon)$ *where* $Re(\lambda_{r_s+1})<0$, $\varepsilon=|\lambda_1|/|\lambda_{r_s+1}|$ *is a small number. We also assume that* $\mathbb{W}\triangleq span\{\phi_i\}_{i=1}^{\infty}$, *that is, operator A is a strong generator of the Sobolev subspace of* $\mathbb{W}$.

Note that $Re(\varepsilon)$ denotes the real part and $O(\varepsilon)$ the order of magnitude of ε. From Assumption 1, we obtain that the Sobolev subspace defined for the infinite dimensional representation of the system, $\mathbb{W}\triangleq\text{span}\{\phi_i\}_{i=1}^{\infty}$, can be partitioned into two other Sobolev subspaces; a slow and a fast one. The slow subspace contains a finite number of basis functions that correspond to slow and possibly unstable modes of x, $\mathbb{W}_s\triangleq\text{span}\{\phi_i\}_{i=1}^{r_s}$. The fast complement subspace contains an infinite number of basis functions that correspond to fast and stable modes of x, $\mathbb{W}_f\triangleq\text{span}\{\phi_i\}_{i=r_s+1}^{\infty}$. Note that there is a time scale separation between the dynamics of the two subsystems; the state of the infinite dimensional system of (8) can be partitioned into a finite dimensional subspace of slow and possibly unstable modes and an infinite dimensional subsystem of stable and fast modes

$$x=x_s+x_f \tag{11}$$

where $x_s=\mathcal{P}x\in\mathbb{W}_s$, $x_f=\mathcal{Q}x\in\mathbb{W}_f$, and $\mathbb{W}=\mathbb{W}_s\oplus\mathbb{W}_f$. The orthogonal integral projection operators are defined as $\mathcal{P}:\mathbb{W}\to\mathbb{W}_s$, $\mathcal{P}=(\cdot,\Phi_s)$ and $\mathcal{Q}:\mathbb{W}\to\mathbb{W}_f$, $\mathcal{Q}=(\cdot,\Phi_f)$.

Then, using the method of weighted residuals based on the set of basis functions, the system of (8) can be presented as an ODE set of vectorized modes in the following form

$$\begin{aligned}\dot{x}_s&=A_sx_s+f_s(x_s,x_f,\theta)+B_su,\ x_s(0)=\mathcal{P}x_0\\ \dot{x}_f&=A_fx_f+f_f(x_s,x_f,\theta)+B_fu,\ x_f(0)=\mathcal{Q}x_0\end{aligned} \tag{12}$$

where $A_s=\mathcal{P}\mathcal{A}=\text{diag}\{\lambda_i\}_{i=1}^{r_s}$, $A_f=\mathcal{Q}\mathcal{A}=\text{diag}\{\lambda_i\}_{i=r_s+1}^{\infty}$, $f_s=\mathcal{P}\mathcal{F}$, $f_f=\mathcal{Q}\mathcal{F}$, $B_s=\mathcal{P}\mathcal{B}$, $B_f=\mathcal{Q}\mathcal{B}$, $\mathcal{P}x(0)=\mathcal{P}x_0$, $\mathcal{Q}x(0)=\mathcal{Q}x_0$.

The fast dynamics of (12) can be represented in the following singular perturbation form

$$\varepsilon\dot{x}_f=\varepsilon A_fx_f+\varepsilon(f_f(x_s,x_f,\theta)+B_fu) \tag{13}$$

where due to Assumption 1, $\varepsilon=|\lambda_1|/|\lambda_{r_s+1}|$ is a small number that indicates the time scale separation between the slow and fast dynamics.[1]

Assumption 2. *The control action is assumed to be bounded and the nonlinear dynamics of the fast and stable subsystem satisfies the Lipschitz condition.*[1]

By assuming bounded control action and Lipschitz condition for $f_f(x_s,x_f,\theta)$, we obtain that $f_f(x_s,x_f,\theta)+B_fu$ does not include a term of $O(\frac{1}{\varepsilon})$. Considering that A_f has negative eigenvalues of $O(1)$ and by setting $\varepsilon=0$, the fast dynamics can be presented as the following locally exponentially stable form

$$\frac{\partial x_f}{\partial\tau}=A_{f\varepsilon}x_f \tag{14}$$

where $\tau=\frac{t}{\varepsilon}$ and $A_{f\varepsilon}=\varepsilon A_f$. Note that $A_{f\varepsilon}$ is of order $O(1)$ in the fast time scale. After a period of time, x_f converges to a ball of radius $O(\varepsilon)$ around zero. Thus, the fast dynamics of x_f can be ignored compared to the slow dynamics of x_s. Accordingly, the process dynamics can be approximated as follows

$$\dot{x}_s=A_sx_s+f_s(x_s,0,\theta)+B_su,\ x_s(0)=\text{P}x_0 \tag{15}$$

Assumption 3. *The PDE system of (1)–(3) and as a result, the slow and fast subsystems of (12) are assumed to be approximately observable and controllable.*[36]

On the basis of the above expositions, we need to solve the eigenvalue problem of (9) and find the set of basis functions to reduce the infinite dimensional system. However, generally it is not possible to solve it when we have complex boundary conditions. The interesting fact is that we cannot find the analytical solution even for a general class of linear systems over miscellaneous domains. Conversely, even if we can compute them analytically, we are not able to directly identify how many of them are enough to capture the dominant dynamic behavior of the system. Note that using large number of basis functions increases the dimensionality of ROM and the controller computational demand. Thus, most of the standard analytical MOR techniques cannot be directly used even for distributed parameter systems described by general linear PDEs. One solution to circumvent this issue is to apply APOD as explained briefly in the next section.

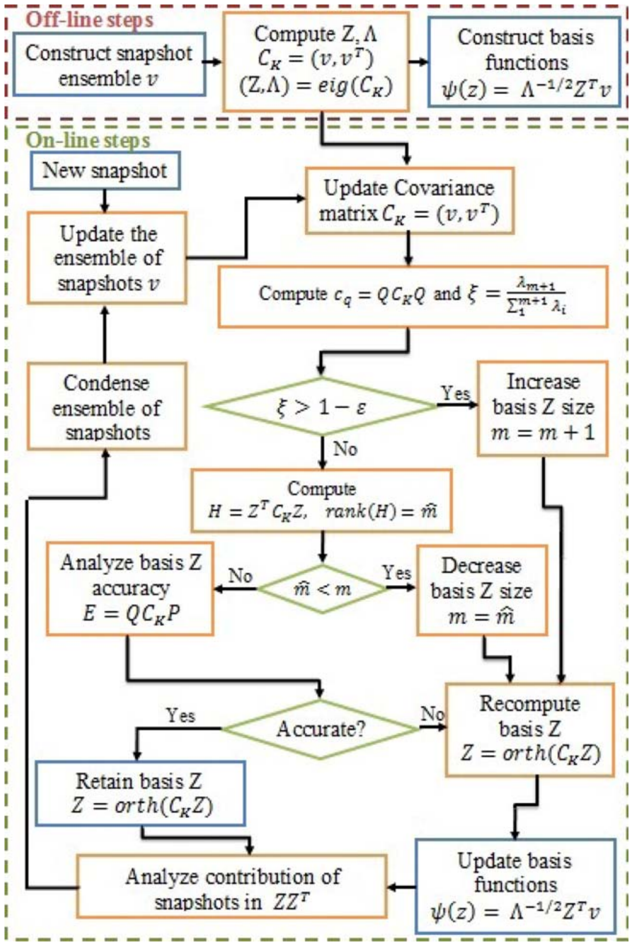

Figure 1. Flow chart of APOD.[21]

Blue denotes the algorithm I/O, green decision steps, and orange computations. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Remark 2. Assumption 1 is always satisfied when the system is linear. It is also automatically satisfied for semilinear systems when we have bounded Lipschitz nonlinearity. The reader may refer to Ref. 37 for semilinear PDE definition.

Remark 3. The proposed method is not only applicable to parabolic systems but also to higher order semilinear systems that satisfy Assumption 1, such as physicochemical systems described by Kuramoto–Sivashinsky equation[19,38,39] and Korteweg de Vries–Burgers equation.[40]

Adaptive Model Order Reduction

Adaptive proper orthogonal decomposition

The flow chart of APOD including its algebraic steps is presented in Figure 1. For brevity, we only summarize the main steps in the flow chart in this article. A detailed description of the offline and online steps of the algorithm and a relevant analysis can be found in Refs. 21,23.

1. Offline steps of APOD
   - Assemble the available snapshots of the system in an ensemble.
   - Compute the covariance matrix of the ensemble.
   - Obtain an initial set of empirical basis functions using POD.[23]
2. Online steps of APOD
   - Analyze the contribution of snapshots in dominant eigenspace and condense the ensemble of snapshots by one.[21]
   - Collect a new snapshot from the distributed measurement sensors and update the ensemble of snapshots.[21]
   - Update the covariance matrix.
   - Compute the projection of eigenspace on slow and fast parts.
   - Increase, decrease, or retain the dimensionality of the approximated dominant eigenspace.[23] If retaining, compute the projection error.
   - If needed, revise the eigenspace and update the empirical basis functions.

Finite dimensional approximation using Galerkin's method

The finite-dimensional approximation of the infinite-dimensional representation of (1)–(3) can be computed using the method of weighted residuals when the set of empirical basis functions are available. Generally, the original state of the PDE system, $\bar{x}(z,t)$, can be described as an infinite weighted summation of a complete vectorized set of basis functions $\Psi(z)$ as follows

$$\bar{x}(z,t) \approx \sum_{k=1}^{r_m} \psi_k(z) a_k(t) \xrightarrow{r_m \to \infty} \bar{x}(z,t) = \sum_{k=1}^{\infty} \psi_k(z) a_k(t) \quad (16)$$

where $a_k(t)$ for $k = 1, \ldots, r_m$ are time varying coefficients known as system modes. The following r_mth order system of ODEs is obtained by substituting (16) in (1)–(3), multiplying the PDE with the weighting functions, $\varphi(z)$, and integrating over the entire spatial domain

$$\begin{aligned} \sum_{k=1}^{r_m} (\varphi_v(z), \psi_k(z)) \dot{a}_k(t) &= \sum_{k=1}^{r_m} (\varphi_v(z), \mathcal{A}\psi_k(z)) a_k(t) \\ &+ (\varphi_v(z), \mathcal{F}(z, \sum_{k=1}^{r_m} \psi_k(z) a_k(t), \theta)) + (\varphi_v(z), b(z)) u, \\ &v = 1, \ldots, r_m \\ y_m &= \sum_{k=1}^{r_m} (s(z), \psi_k(z)) a_k(t) \end{aligned} \quad (17)$$

The type of weighted residual method can be determined by the weighting functions in the above equation. The method reduces to Galerkin projection when the weighting functions, $\varphi(z)$, and the basis functions, $\Psi(z)$, are the same. Then, (17) can be summarized as

$$\begin{aligned} \dot{a} &= Aa + f(a, \theta) + Bu \\ y_m &= Sa \end{aligned} \quad (18)$$

where $A^{r_m \times r_m}$, $B^{r_m \times l}$, and $S^{w \times r_m}$ are constant matrices and f is a nonlinear smooth vector function of the modes that can be described based on the comparison between (17) and (18). From Lipschitz condition of $\mathbf{f}$, we obtain that f satisfies a local Lipschitz condition. Note that this assumption can be concluded from the local Lipschitz property of the nonlinear part of the original system of (8) in the Sobolev subspace for special cases.

Remark 4. The main reason for applying APOD to recursively revise the set of empirical basis functions is the appearance of new trends in the process dynamics during process

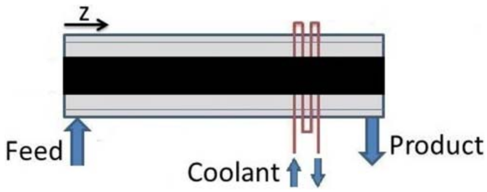

Figure 2. Catalytic chemical reactor.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

evolution. Such new trends make the empirical basis functions and ROM inaccurate. In that case, APOD algorithm revises the basis functions and modifies the ROM. Then following the ROM revisions, the output feedback control structure is redesigned to retain relevancy. Such "corrections" will always be repeated at revision times to construct an accurate basis for controller design and enforce closed-loop stability.[21,41]

Remark 5. As the empirical basis functions of the system are periodically revised, the ROM of (18) we obtain is a switching system, that is, the structure and dimensionality of the system approximation changes as process evolves. Furthermore, the designed adaptation and controller laws will also be revised as well at each switching.[21,27]

Application to Thermal Regulation in a Catalytic Chemical Reactor

In this section, the adaptive model order reduction (AMOR) and control design is applied to a $1D$ diffusion-reaction process. First, we describe the model of thermal dynamics in a catalytic chemical reactor. Then, we apply APOD and Galerkin's method to construct the ROM that is the basis for controller synthesis. Finally, we design an adaptive controller to regulate the temperature dynamics of the catalytic reactor and present the results.

Note that the main contribution of this article is toward the AMOR component which facilitates the implementation of adaptive control approaches to distributed systems with unknown parameters. Adaptive control is a well-known control approach as we discussed in "Introduction" section. We are using a Lyapunov-based adaptive method to design the controller. For brevity reasons, we do not present the basics and theory of adaptive control approach which can be found in Refs. 2,4–6.

Reactor description

Consider an elementary exothermic reaction A → B taking place on a long, thin rod in a catalytic chemical reactor that is presented in Figure 2. As the reaction is exothermic, a coolant is used to remove heat from the reactor. To model temperature dynamics inside the catalytic rod, we assume constant density, heat capacity, and constant conductivity of the rod, and constant temperature at both ends of the rod. We also assume that the reactant A is present in excess and thus can assume constant reactant concentration. The mathematical model that describes the spatiotemporal thermal dynamics inside the catalytic rod is presented by the following semilinear parabolic PDE

$$\frac{\partial \bar{x}}{\partial t}=D\frac{\partial^2 \bar{x}}{\partial z^2}+\beta_T(\mathrm{e}^{-\varepsilon/(1+\bar{x})}-\mathrm{e}^{-\varepsilon})+\beta_U(b(z)u(t)-\bar{x}) \tag{19}$$

subject to boundary and initial conditions

$$\bar{x}(0,t)=0,\ \bar{x}(\pi,t)=0,\quad \bar{x}(z,0)=\bar{x}_0(z) \tag{20}$$

where $\bar{x}$ denotes the dimensionless temperature of the catalytic rod that is defined over the spatial domain of $[0,\pi]$ and $z\in[0,\pi]$ is the spatial coordinate, D is the dimensionless diffusion coefficient and β_T denotes the dimensionless heat of reaction; γ stands for the dimensionless activation energy, and the parameter β_U denotes the dimensionless heat-transfer coefficient. The vector of control variables is denoted by $u(t)$ and $b(z)$ describes the spatial distribution of the actuator, for example, point actuation could be defined using standard Dirac delta. Figure 3 presents the open-loop spatiotemporal profile of the system state and temporal profile of its 2-norm when the nominal values of the system unknown parameters are $D=1$, $\beta_T=16$, and $\gamma=2$ and the dimensionless heat-transfer coefficient of the system is $\beta_U=2$. The initial dimensionless temperature of the catalytic rod is considered as a periodic profile with the average value of $\bar{x}_{0,\mathrm{avg}}=0.5$. The hot spot can be observed easily at the middle point of the catalytic rod. The control problem thus becomes regulating the catalyst temperature at a uniform temperature of $\bar{x}=0$.

To address the control problem, we consider the linearization of the PDE system of (19) around the spatially uniform unstable steady state of $\bar{x}(z,t)=0$. It has the following form

$$\frac{\partial \bar{x}}{\partial t}=D\frac{\partial^2 \bar{x}}{\partial z^2}+(\beta_T\mathrm{e}^{-\gamma}\gamma-\beta_U)x+\beta_U b(z)u(t) \tag{21}$$

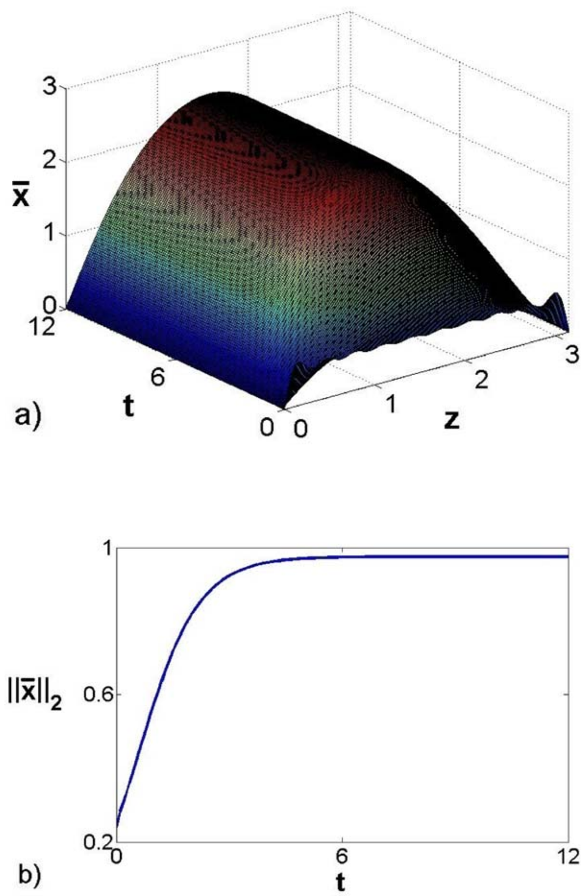

Figure 3. Open-loop (a) spatiotemporal profile of the system state and (b) temporal profile of its 2-norm.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

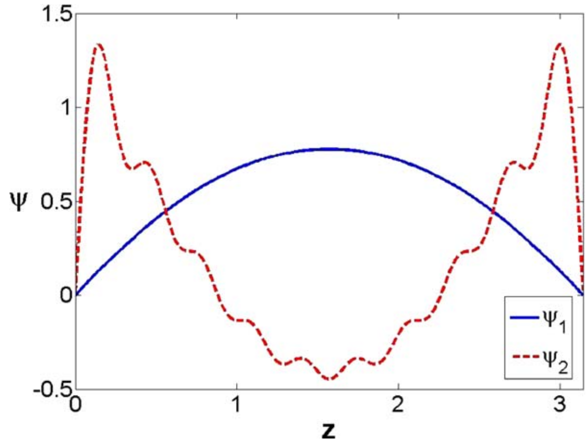

Figure 4. The initial dominant empirical basis function (obtained from offline APOD part).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

We assume that we do not know the reaction and diffusion parameters inside the reactor, that is, the parameters β_T, γ, and D that indicate heat of reaction, activation energy, and diffusion coefficient, respectively, are unknown. The only parameter of the system that is known is the dimensionless heat-transfer coefficient, $\beta_U = 2$. The linearized PDE system of (21) can be represent in the form of reaction and diffusion uncertainty as follows

$$\frac{\partial \bar{x}}{\partial t} = \theta_1 \frac{\partial^2 \bar{x}}{\partial z^2} + \theta_2 x + \beta_U b(z)u(t) \tag{22}$$

where $[\theta_1 \ \theta_2]^T = [D \ \ \beta_T e^{-\gamma}\gamma - \beta_U]^T$.

Four continuous point measurement sensors are assumed to be placed uniformly across the domain of the process, $L_s = [\pi/5, 2\pi/5, 3\pi/5, 4\pi/5]$, where $y_m(L_s) = \Psi^T(L_s)a$ and $\Psi = [\psi_1 \, \psi_2 \, \cdots \, \psi_{r_m}]^T$, and we consider the availability of only one point actuator at $L_a = 3\pi/4$, where the corresponding spatial distribution functions at the location for point measurements and actuation is considered as the Dirac delta function, $\delta(z-L)$. The profiles of the rod temperature is also assumed to be accessible every $\delta t = 1s$. The control objective is the regulation of the temperature dynamics in the presence of unknown reaction and diffusion parameters at the unstable steady state of $\bar{x}(z,t) = 0$ based on AMOR.

Remark 6. Considering the local linearization as the basis to design the adaptive control structure for governing nonlinear systems is a promising approach that has been widely applied in the literature.[4–6] We consider this approach to simplify the adaptation law and adaptive controller derivations.

Remark 7. Note that the linearization error between model and nonlinear process is also circumvented using a linear model-based controller design whose parameters adaptively change during process evolution.

AMOR via APOD and Galerkin's method

Even for the specific process we cannot compute the dominant basis functions analytically due to the unknown diffusivity coefficient. We used 20 snapshots and the offline step of APOD to find an initial set of empirical basis functions. These snapshots are generated during the time period of $t \in [0, 2]$ for open-loop process, $u(t) = 0$. Using offline APOD to compute the basis of the initial ensemble of snapshots, we obtain two empirical basis function for $\bar{x}$ which captures 0.999 of the total energy of the ensemble. The initial dominant empirical basis function is presented in Figure 4. The first dominant basis function identifies the dominant dynamics of the open-loop system and the second one is an artifact of the specific initial single trajectory in the absence of control action.

The online APOD adaptively revises the empirical basis functions every $\delta t = 1s$ to keep them accurate. Note that the snapshots needed in Galerkin's method to construct the linear ROMs are obtained from the original nonlinear process. If we consider $\{\psi_i\}_{i=1}^{r_m}$ is the set of empirical basis functions between the revisions, we can approximate the state of the PDE system of (22) as $\bar{x}(z,t) \approx \sum_{i=1}^{r_m} a_i(t)\psi_i(z)$. By substituting this approximation in the PDE of (22), we obtain

$$\sum_{i=1}^{r_m} \dot{a}_i \psi_i = \theta_1 \sum_{i=1}^{r_m} a_i \frac{d^2\psi_i}{dz^2} + \theta_2 \sum_{i=1}^{r_m} a_i \psi_i + \beta_U b(z)u \tag{23}$$

Then, the ROM can be constructed using Galerkin's method as follows

$$\dot{a}_j = \theta_1 \sum_{i=1}^{r_m} a_i \int_0^\pi \frac{d^2\psi_i}{dz^2}\psi_j dz + \theta_2 a_j + \beta_U \left(\int_0^\pi b(z)\psi_j dz \right) u, \qquad j = 1, \ldots, r_m \tag{24}$$

The above equation can be summarized as the following vectorized form

$$\dot{a} = (\theta_1 A + \theta_2 I)a + Bu \tag{25}$$

where $[A]_{j,i} = (\psi_j(z), \frac{d^2\psi_i(z)}{dz^2})$, $[Bu]_j = (\psi_j(z), b(z))u$. The ODE system of (25) is the basis to synthesize the controller. Note that the resulting system approximation of (25) is a switching system due to the form and dimensionality changes during process evolution. Thus, the stability of the switching system under the control actions must be proven via Lyapunov and hybrid system stability arguments.[42,43]

Derivation of APOD-based adaptive control structure

In this section, we utilize the ROM of (25) in designing adaptive output feedback controllers for the semilinear parabolic PDE system of (19). The controller is synthesized based on continuous point measurements available from restricted number of sensors (four sensors). We use a static observer of the following form

$$\tilde{a} = (\Psi(L_s)\Psi^T(L_s))^{-1}\Psi(L_s)y_m(L_s) \tag{26}$$

to estimate the state of the ODE system of (25) due to the unavailability of full state measurements assuming that the number of point measurement sensors is equal to or greater than r_m.

We consider the following control Lyapunov function (CLF)

$$V_c = \frac{\zeta}{2} a^T a \tag{27}$$

where $\zeta > 0$ is a adjusting parameter which modifies the CLF at the revisions. By substituting (25) in the time derivative of the CLF, we obtain

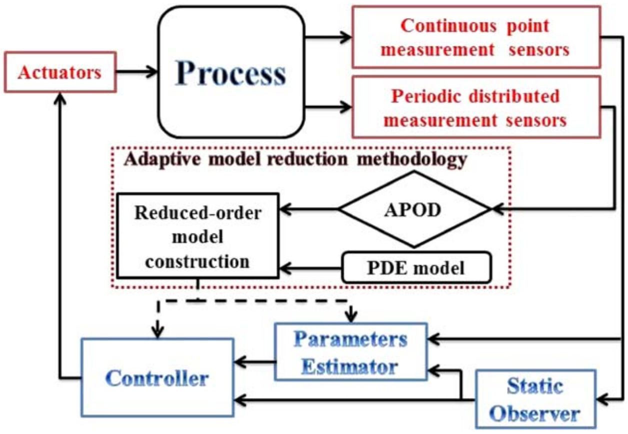

Figure 5. Process operation block diagram under proposed control structure.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

$$\dot{V}_c = \zeta a^T \dot{a} = \zeta a^T((\theta_1 A + \theta_2 I)a + Bu) \tag{28}$$

Then, by considering

$$\dot{V}_c = -\rho a^T a < 0 \tag{29}$$

the control structure is derived as a function of unknown system parameters as follows

$$u = -B^{-1}\left(\frac{\rho}{\zeta}a + (\theta_1 A + \theta_2 I)a\right) \tag{30}$$

where ρ is a positive number that indicates the regulation rate.

We define the closed-loop Lyapunov function as a combination of the CLF and the identification Lyapunov function

$$V = \frac{\zeta}{2}a^T a + \frac{1}{2}\tilde{\theta}_1{}^2 + \frac{1}{2}\tilde{\theta}_2{}^2 \tag{31}$$

where $\tilde{\theta}_1 = \hat{\theta}_1 - \theta_1$, $\tilde{\theta}_2 = \hat{\theta}_2 - \theta_2$, and $\hat{\theta}_1$, $\hat{\theta}_2$ are the estimations of the unknown parameters of θ_1 and θ_2, respectively.

By substituting (25) in the time derivative of the closed-loop Lyapunov function and assuming that θ_1 and θ_2 do not vary continuously, we conclude

$$\begin{aligned}\dot{V} &= \zeta a^T \dot{a} + \tilde{\theta}_1 \dot{\hat{\theta}}_1 + \tilde{\theta}_2 \dot{\hat{\theta}}_2 \\ &= \zeta a^T((\theta_1 A + \theta_2 I)a + Bu) + \tilde{\theta}_1 \dot{\hat{\theta}}_1 + \tilde{\theta}_2 \dot{\hat{\theta}}_2\end{aligned} \tag{32}$$

The controller formula in the presence of unknown parameters is obtained by the applying certainty equivalence principle[4] to the control structure of (30)

$$u = -B^{-1}\left(\frac{\rho}{\zeta}a + (\hat{\theta}_1 A + \hat{\theta}_2 I)a\right) \tag{33}$$

Then, by considering (32) and (33) we can obtain the adaptation laws

$$\begin{aligned}\dot{\hat{\theta}}_1 &= a^T A a \\ \dot{\hat{\theta}}_2 &= a^T a\end{aligned} \tag{34}$$

where

$$\dot{V} = -\rho a^T a < 0 \tag{35}$$

thus the closed-loop system is locally asymptotically stable in the Lyapunov sense.

The control and adaptation laws of (33) and (34) are redesigned when the ROM is revised by APOD. Stability theorems of hybrid systems are then required to prove that the resulting switching controlled system remains stable at revisions. For switching system stability analysis, the multiple CLFs in the form of (27) must be considered. The negative time derivative of the multiple CLFs, described in (35), guarantees stability of the switching system (Theorem 3.2 in Ref. 43) when the following condition is also satisfied[42,43]

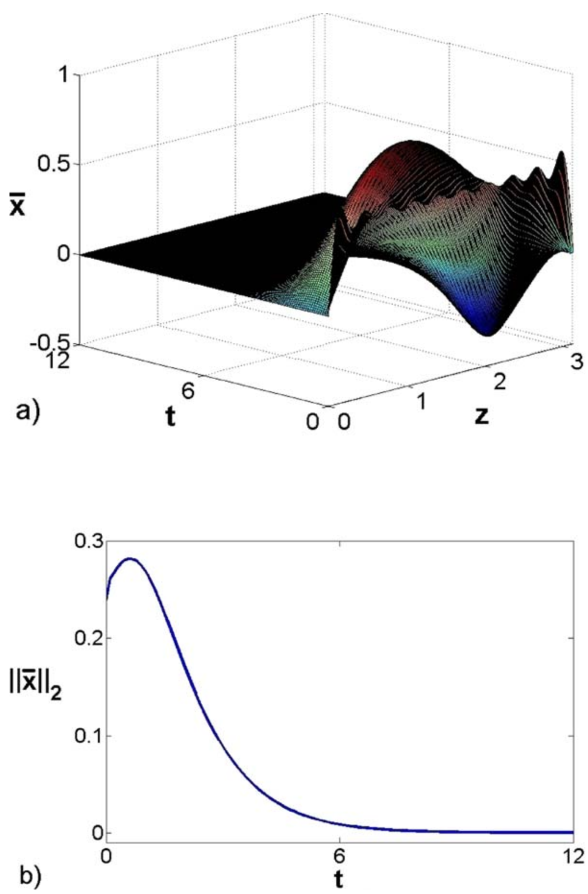

Figure 6. Closed-loop (a) spatiotemporal profile of the system state and (b) temporal profile of its 2-norm.

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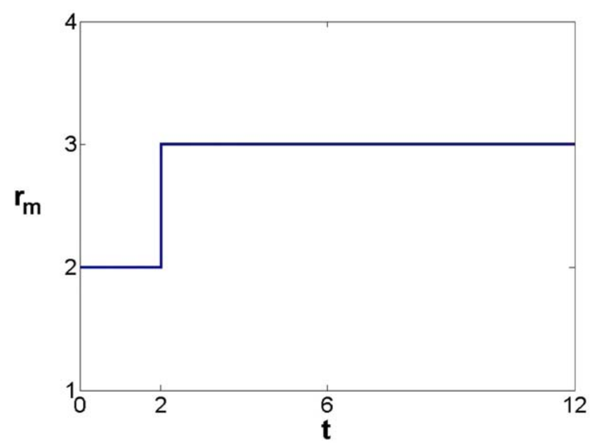

Figure 7. Number of empirical basis functions.

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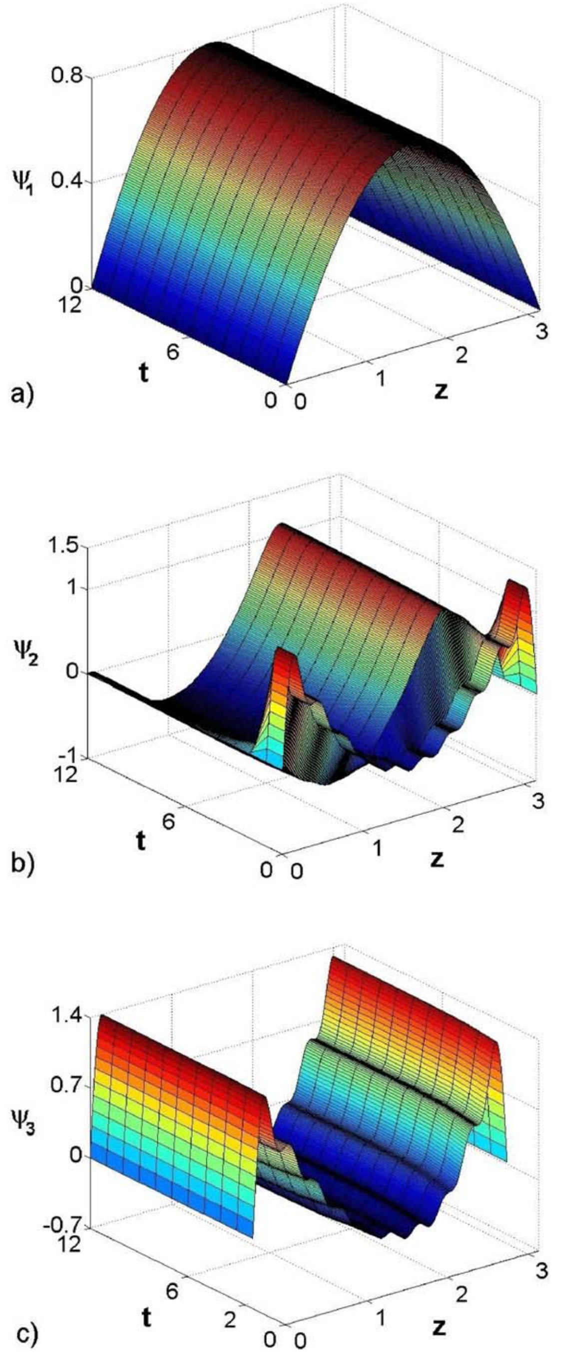

Figure 8. Temporal profile of the (a) first, (b) second, and (c) third empirical basis functions.

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$$V_c(\tilde{a}(t_k)) < V_c(\tilde{a}(t_{k-1})) \tag{36}$$

where $k > 1$ and $V_c(\tilde{a}(t_k))$ corresponds to the value of CLF at the beginning of time interval $[t_k, t_{k+1}]$. The CLF may possibly increase during dimensionality changes of the ROM, furthermore, the value of ζ must be chosen appropriately in the closed-loop process. The following equation is applied to update the value of ζ as needed

$$\zeta = \eta \frac{\tilde{a}^T(t_{k-1})\,\tilde{a}(t_{k-1})}{\tilde{a}^T(t_k)\,\tilde{a}(t_k)} \tag{37}$$

where $\eta < 1$. Alternatively, we initialize ζ at value $\zeta_0 = 1$, and re-evaluate it using (37) only when the criteria of (35) is violated.

Figure 5 shows the block diagram of closed-loop process under the proposed control structure. The control performance of the system of (1)–(3) directly depends on the accuracy of its ROM. Using APOD methodology, the ROM will be adaptively revised at certain time instants to remain accurate. Then, the ROM structure and dimensionality change during process evolution. As a result, when the ROM switches the controller will be redesigned as well.

Remark 8. The static observer requirements of available number of continuous point measurement sensors being supernumerary to the dimension of the ROM can be circumvented using dynamic observer synthesis which conceptually requires only one measurement output.[25,28,44]

Remark 9. To design the controller in the form of (33) we assume that the number of control actuators is equal to r_m and the inverse of B exists. To satisfy such conditions, the number of actuators may be decreased or increased during the process evolution if the dimensionality of the ROM changes. We may avoid activating and deactivating actuators during such changes by using more than r_m actuators and employing the right pseudoinverse of B, $B^{\perp} = B^T(BB^T)^{-1}$ in (33), instead of B^{-1}.

Remark 10. The adaptive control of nonlinear distributed parameter systems in the presence of time-varying unknown parameters using dynamic observer designs is the current

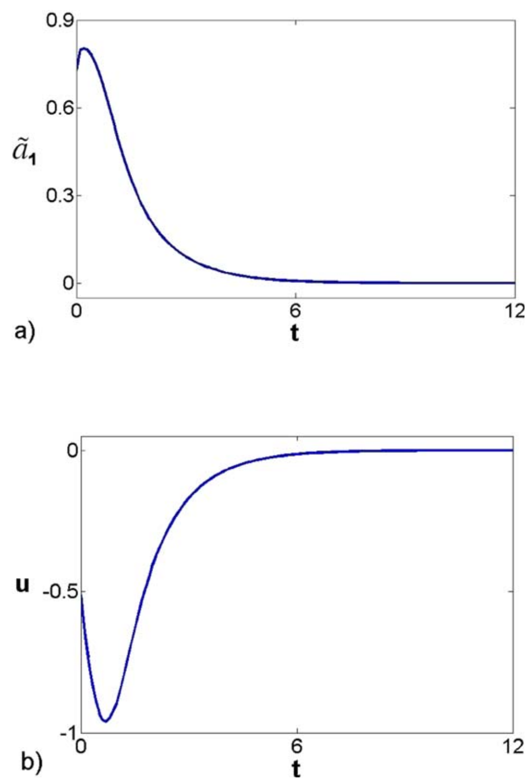

Figure 9. Temporal profile of (a) the estimated dominant mode of the system and (b) the required control action.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

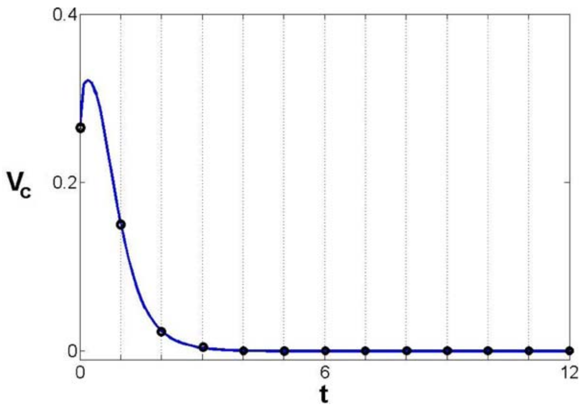

Figure 10. Temporal profile of the CLF.

The black circles show the CLF values at the ROM revisions. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

subject of authors' research and will be presented in future publications.

Closed-loop analysis

We present the closed-loop simulation results for the catalytic chemical reactor in the presence of unknown diffusion and reaction parameters. We studied two cases with the same controller structure and parameters; (1) constant diffusion-reaction parameters and (2) time-varying the diffusion-reaction parameters as process evolves. In both cases, we use $\rho = 2$ to adjust the rate of asymptotic decrease in the Lyapunov function.

Constant Diffusion-Reaction Parameters. In this section, the closed-loop simulation results are presented for thermal dynamic regulation of the catalytic reactor when the unknown diffusion-reaction parameters are constant and do not change during the process evolution. Figure 6 illustrates the closed-loop process profile and its 2-norm. We can easily observe that the adaptive controller successfully regulates the system of (19) and (20) at the unstable steady state of $\bar{x}(z,t) = 0$.

Figure 7 shows the change in the number of empirical basis functions required to capture the dominant dynamics of the system during the process evolution. The number of initial empirical basis functions was two and it increased to three as the process evolved. Figure 8 shows the temporal profiles of the first, second, and third empirical basis functions. The dominant empirical basis functions were updated to accurately capture the process behavior when new trends appeared. The temporal profiles of the estimated dominant mode of the system and the required control action are presented in Figure 9. The system dominant mode and the control action converged to the steady-state value of zero without chattering and any sharp changes. The effects of changes in the unknown parameters of the system can be observed in the temporal profiles of the 2-norm, dominant estimated mode, and control action at $t = 5s$. When the actuator is activated, the process behavior significantly changes and the effect of the actuator on the DPS is now captured by the second basis function which has a maximum at $z = 3\pi/4$ where the actuator is located. We see that APOD adapts to this change and a new basis is captured where the closed-loop dynamics.

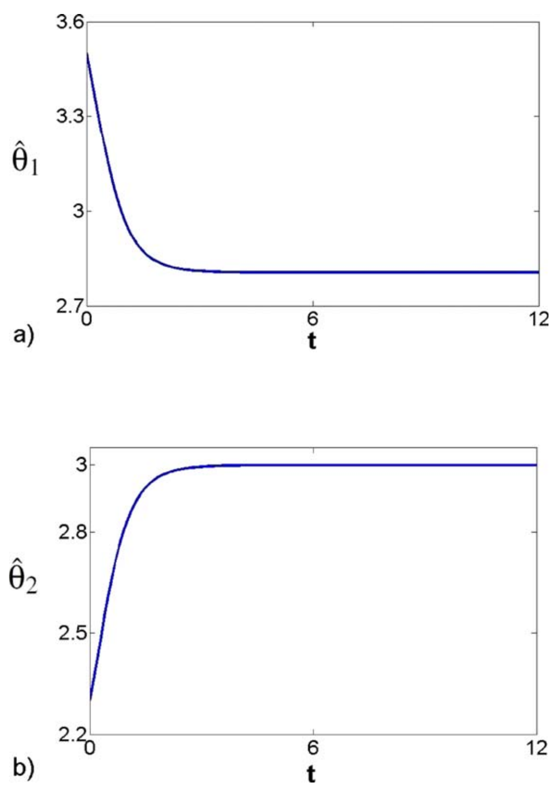

Figure 11. Temporal profile of (a) $\hat{\theta}_1$ and (b) $\hat{\theta}_2$.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

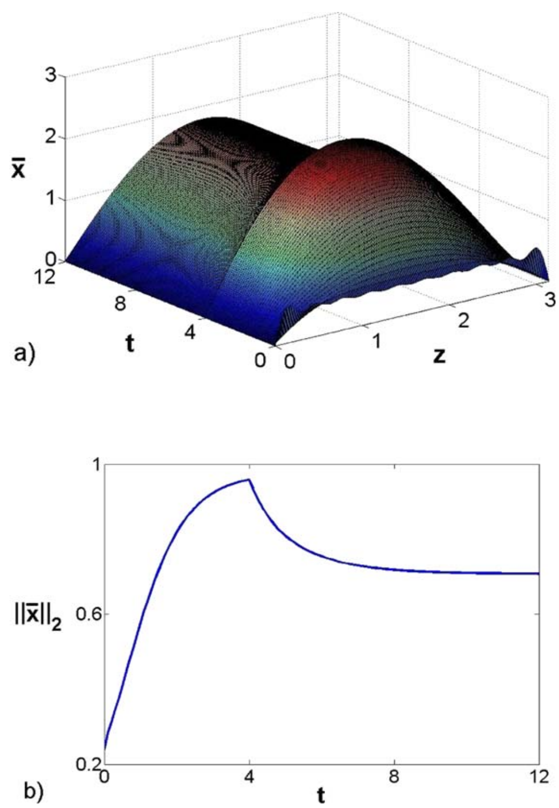

Figure 12. Open-loop (a) spatiotemporal profile of the system state and (b) temporal profile of its 2-norm in the presence of unknown parameters changes.

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Figure 10 presents the temporal profile of the CLF for $\zeta = \zeta_0 = 1$. The black circles indicate the CLF values at the revisions. We observe that the CLF values at the revisions decreased as time evolves which guarantees the switching system stability. In such case, we did not need to re-evaluate ζ because the criteria of (35) was always valid during process evolution. Figure 11 presents the dynamics of the estimated unknown parameters from the adaptation law. We observe that the estimated parameters converged the steady-state values of $\hat{\theta}_1 = 2.8$ and $\hat{\theta}_2 = 3$ when the nominal values of the system unknown parameters are $\theta_1 = 1$ and $\theta_2 = 2.33$. We may observe that the adaptation law cannot estimate the unknown parameters due to the lack of persistent excitation needed for complete system identification. Note that the system identification is not the objective of the proposed control method.

Time-Varying Diffusion-Reaction Parameters. In this section, we present the simulation results in the presence of unknown diffusion and reaction parameters that change from $D = 1$, $\beta_T = 16$ and $\gamma = 2$ to $D = 3$, $\beta_T = 30$, and $\gamma = 3$ at $t = 4s$. Figure 12 shows the open-loop process spatiotemporal profile and the temporal profile of its 2-norm, when the controller was inactive. We observe a significant change in the system dynamics due to the parametric change.

The closed-loop system profile and its 2-norm under the impact of the proposed control structure is presented in Figure 13, where the adaptive controller stabilized the process at the unstable steady state of $\bar{x}(z, t) = 0$. Finally, we illustrate the temporal profiles of the estimated dominant mode and the

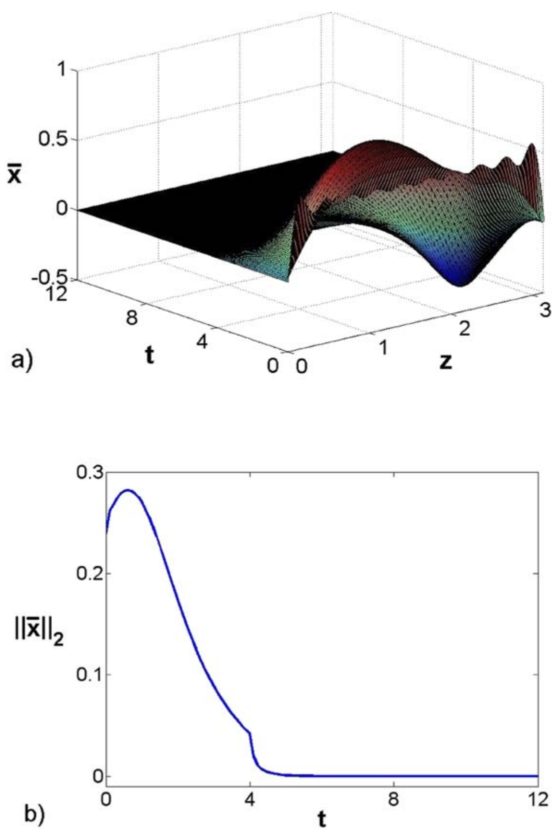

Figure 13. Closed-loop (a) spatiotemporal profile of the system state and (b) temporal profile of its 2-norm in the presence of unknown parameters changes.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

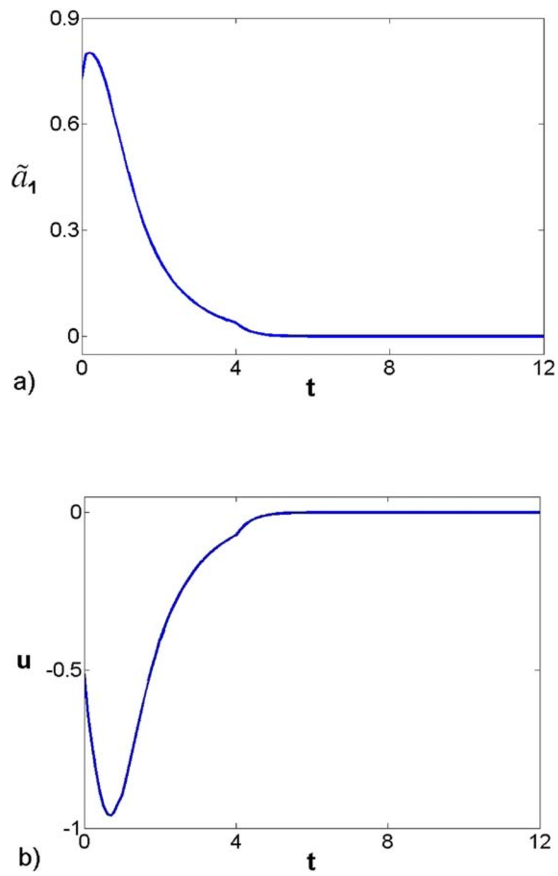

Figure 14. Temporal profile of (a) the estimated dominant mode of the system and (b) the required control action in the presence of unknown parameters changes.

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control action in Figure 14. The convergence of the dominant mode and control action to zero without any chartering indicates the effectiveness of the proposed adaptive control method.

The effects of the changes in the unknown parameters of the system can be observed in the temporal profiles of the closed-loop 2-norm, dominant estimated mode and control action at $t = 4s$. Due to the effective performance of the two layer adaptation we do not observe significant variations in the closed-loop system when the process unknown parameters change.

The success of the designed controller in regulating the process is due to the dominant eigenspace (hence the ROM, the adaptation, and the control laws) being updated as the process traverses through different regions of the state space during closed-loop operation. During the closed-loop process operation, when new trends appeared the dominant empirical basis functions were updated to accurately capture the process behavior.

Conclusions

The control problem of dissipative distributed parameter systems with unknown parameters which are modeled by semilinear parabolic PDEs is investigated via MOR in this manuscript. The unknown parametric changes have a significant effect on MOR approaches and the resulting ROMs. We use the combination of APOD and Galerkin's method to construct the ROM that is the basis for adaptive controller design. The proposed control strategy is illustrated on thermal regulation in a catalytic chemical reactor in the presence of constant and time-varying unknown parameters.

Acknowledgment

Financial support from the National Science Foundation, CMMI Award # 13-00322 is gratefully acknowledged.

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Manuscript received Aug. 26, 2014, and revision received Jan. 30, 2015.